On the Convergence of Day-Ahead and Real-Time Electricity Markets

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Outline

1. **Motivation: Role of Optimization and High-Performance Computing**

2. **Resolution Inconsistency in Day-Ahead & Real-Time Markets**

3. **Stochastic Optimization**

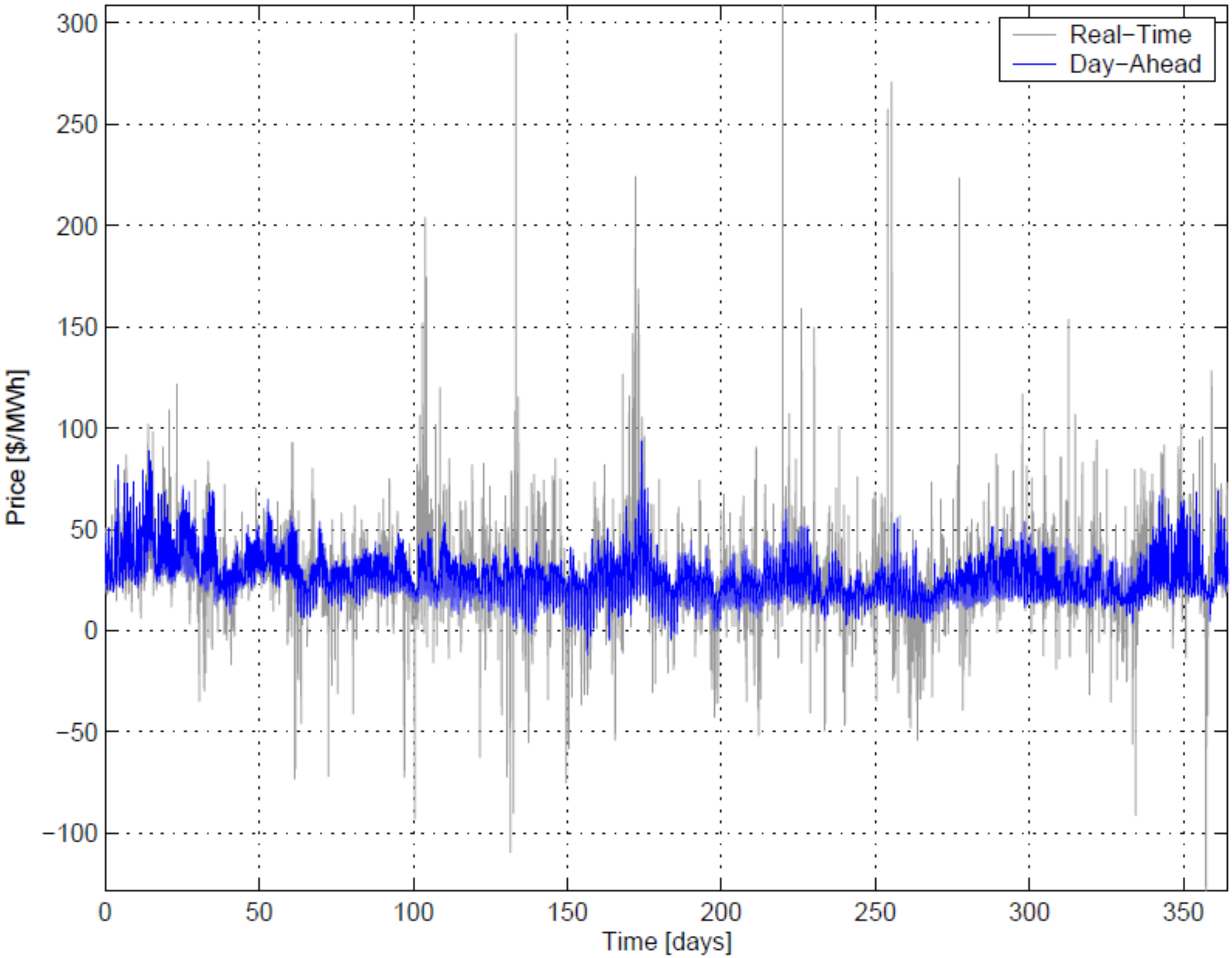
4. **Dynamic Market Stability**

5. **Conclusions and Open Questions**

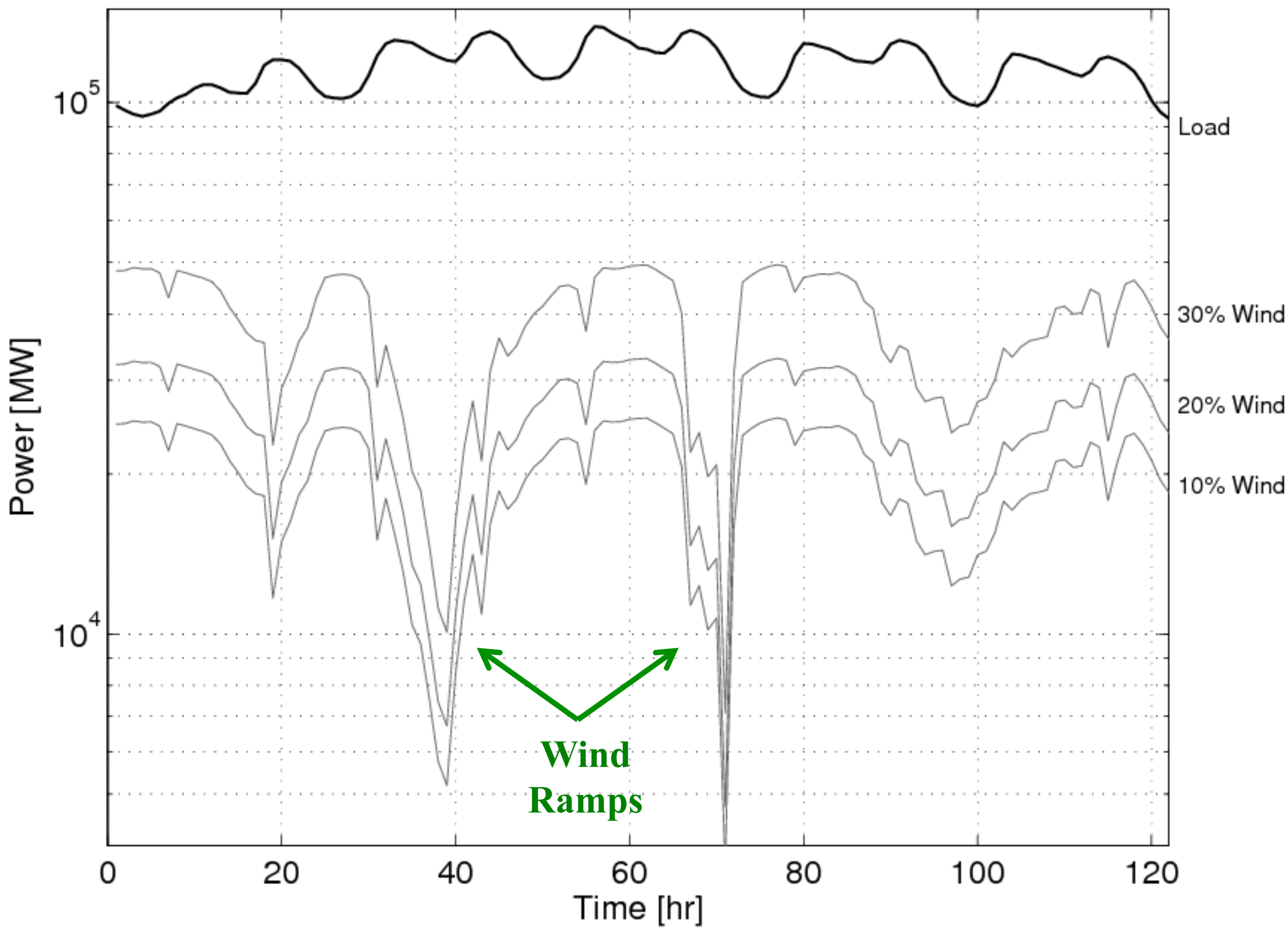
1. Motivation

Market Volatility

Prices at Illinois Hub, 2009



Motivation



Volatility Leads to <u>Uneven Distribution of Welfare</u> and Induces Manipulation

How to Predict and Control Volatility?

2. Resolution Inconsistency in Day-Ahead & Real-Time Markets

Unit Commitment and Economic Dispatch

Unit Commitment

Solved Every 24 Hours, Resolution 1 Hour, <u>Horizon 24-72 Hr</u>

Large-Scale MILP - O(10^5) Continuous, O(10^3) Integer

Economic Dispatch

Solved Every 5 Min, Resolution 5 Min, <u>Horizon 1-2 Hr</u>

Large-Scale LP/QP - O(10^5-10^6) Continuous

$$\min \sum_{k=\ell}^{\ell+N} \sum_{j \in \mathcal{G}} c_j \cdot G_{k,j}$$

$$\text{s.t. } G_{k+1,j} = G_{k,j} + \Delta G_{k,j},\ k \in \mathcal{T}, j \in \mathcal{G}$$

$$\sum_{(i,j)\in\mathcal{L}_j} P_{k,i,j} + \sum_{i\in\mathcal{G}_j} G_{k,i} = \sum_{i\in\mathcal{D}_j} D_{k,i},\ k \in \mathcal{T}, j \in \mathcal{B}$$

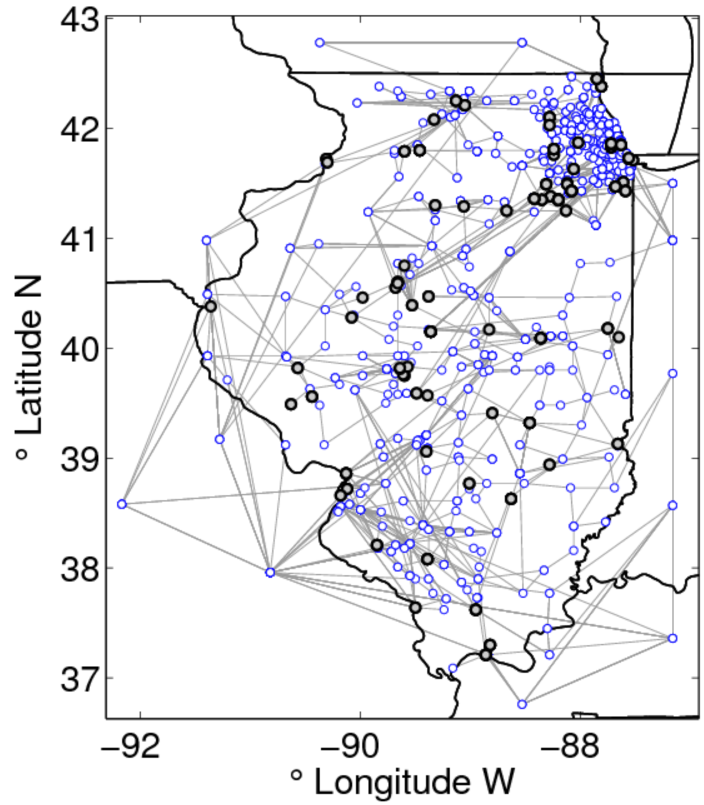
$$P_{k,i,j} = b_{i,j}(\theta_{k,i} - \theta_{k,j}), k \in \mathcal{T}, (i,j) \in \mathcal{L}$$

$$0 \le G_{k,j} \le G_j^{max},\ k \in \mathcal{T}, j \in \mathcal{G}$$

$$0 \le \Delta G_{k,j} \le \Delta G_j^{max},\ k \in \mathcal{T}, j \in \mathcal{G}$$

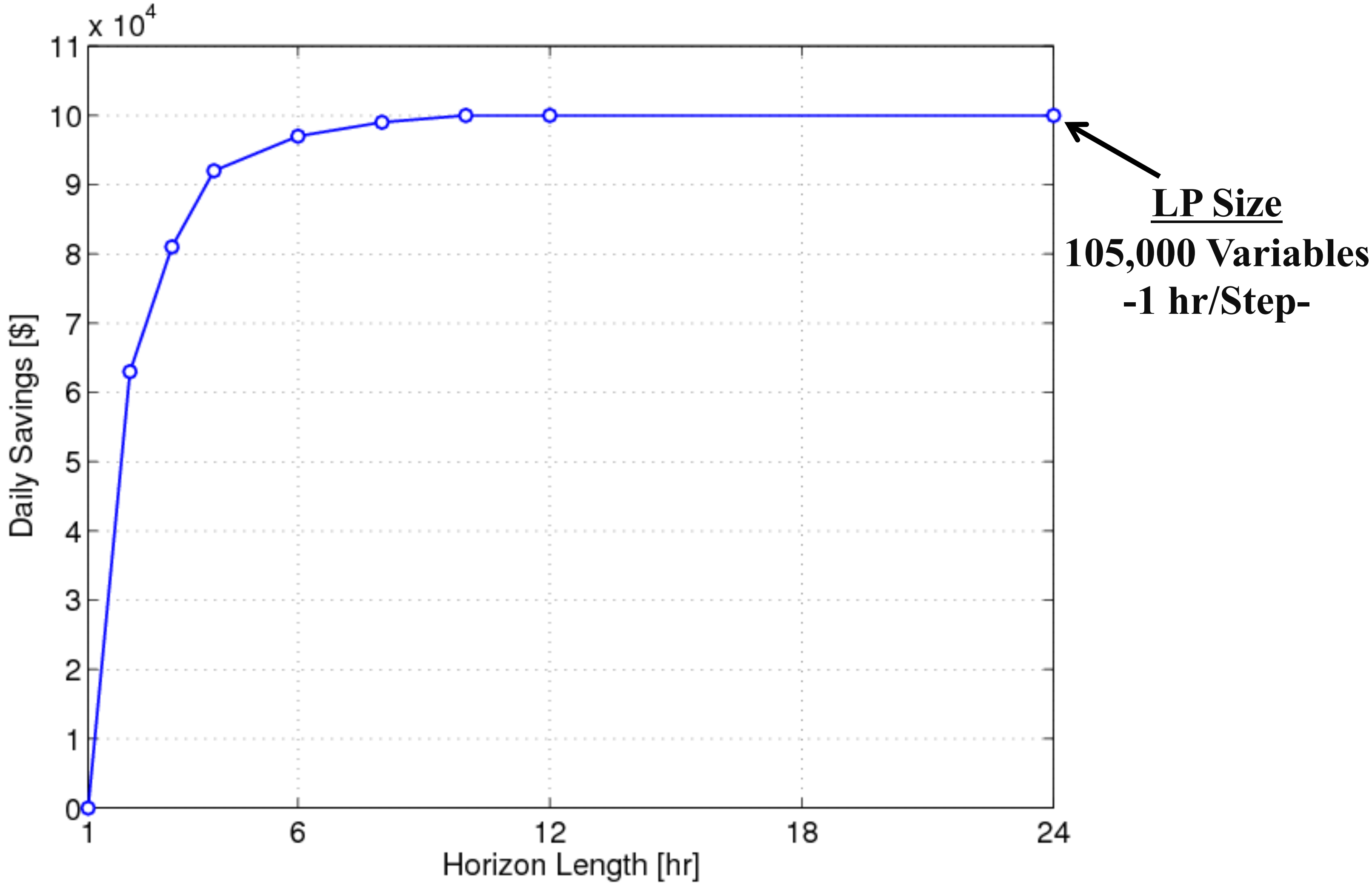
$$|P_{k,i,j}| \le P_{i,j}^{max},\ k \in \mathcal{T}, (i,j) \in \mathcal{L}$$

$$|\theta_{k,j}| \le \theta_j^{max},\ k \in \mathcal{T}, j \in \mathcal{B}$$



Benchmark System –Illinois- 1900 Buses, 2538 Lines, 870 Loads, and 261 Generators

Increasing Horizon of Economic Dispatch



Increasing Horizon Increases Market Efficiency – $O($10^8$) Savings/Yr

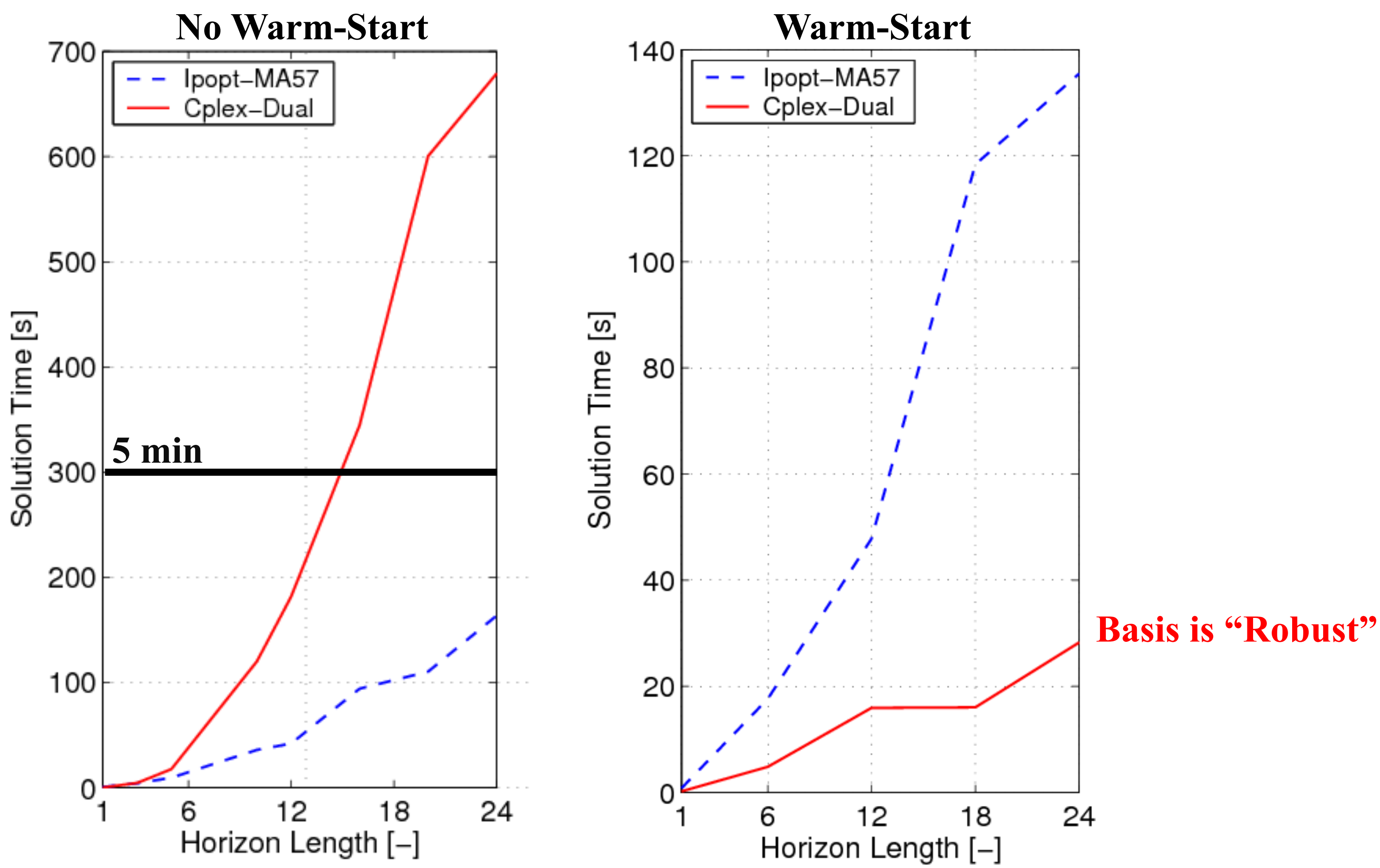
Short Horizons Lead to More Frequent Active Ramps

Savings Constrained by Solution Time -Desired 5 Min- (5 Min Resolution = 2,100,000 Variables)

Increasing Horizon of Economic Dispatch

Linear Algebra: Computational Performance *Z., Botterud, Constantinescu & Wang, 2010*

IPOPT- Symmetric KKT Matrix (MA57) vs. CPLEX-Simplex – Basis Factorization/Updates



Existing Solvers Not Capable of Dealing with High-Resolution Problems

Hybrid Strategy (5 Min Solution Time) - 20 Hr Foresight, 5 Min/Step, 1×10^6 Variables

3. Stochastic Optimization

Stochastic Market Clearing

Claim: StochOpt Improves Convergence of DA and RT Markets *Z. Anitescu 2011*

- Can Anticipate RT Market Recourse and Makes DA Prices Robust

Deterministic Clearing

$$\min_q \quad \mathbf{1}_c^T c(q) \quad \textbf{DA}$$
$$\text{s.t.} \quad \mathbf{M}\cdot q \geq \bar{d} \qquad (p^D \geq 0)$$
$$\underline{q} \leq q \leq \bar{q}$$
$$-r \leq \mathbf{\Pi}\cdot q \leq r$$

$\bar{q}, \bar{p}^D$ **Clearing Signals**

RT Recourse

$$\min_{\delta q(d)} \mathbb{E}_d \left[\mathbf{1}_c^T c(\bar{q} + \delta q(d))\right].$$

Stochastic Clearing

$$\min_q \quad \mathbf{1}_c^T c(q) + \min_{\delta q(d)} \mathbb{E}_d \left[\mathbf{1}_c^T c(q + \delta q(d))\right]$$
$$\text{s.t.} \quad \mathbf{M}\cdot q \geq \bar{d} \qquad (p^D \geq 0)$$
$$\underline{q} \leq q \leq \bar{q}$$
$$-r \leq \mathbf{\Pi}\cdot q \leq r$$
$$\mathbf{1}^T(q + \delta q(d)) \geq d$$
$$\underline{q} \leq q + \delta q(d) \leq \bar{q}$$
$$-r \leq \mathbf{\Pi}\cdot(q + \delta q(d)) \leq r.$$

Theorem: $\min_{\delta q(d)} \mathbb{E}_d \left[\mathbf{1}_c^T c(q + \delta q(d))\right] \leq \min_{\delta q(d)} \mathbb{E}_d \left[\mathbf{1}_c^T c(\bar{q} + \delta q(d))\right]$

Theorem: $\min_{\delta q(d)} \mathbb{E}_d \left[\mathbf{1}_c^T c(q(r_1) + \delta q(d))\right] \leq \min_{\delta q(d)} \mathbb{E}_d \left[\mathbf{1}_c^T c(q(r_2) + \delta q(d))\right], \; r_1 \geq r_2$

Implications: **- Real-Time Market Efficiency Under StochOpt Is Higher**

- Increasing Ramping Capacity Increases Efficiency

Parallel Stochastic Optimization

High-Performance Computing for Stochastic Optimization

1st Stage — **Day-Ahead**

2nd Stage — **Real-Time**

$$
\begin{array}{lllllll}
 & \min & f(x) + \frac{1}{S}\sum_{i=1}^{S} g_s(y_s) & & & & \\
s.t. & & & & & & \\
 & A_0 x & +B_0 y_0 & & & & = b_0 \\
 & A_1 x & & +B_1 y_1 & & & = b_1 \\
 & A_2 x & & & +B_2 y_2 & & = b_2 \\
 & \vdots & & & & \ddots & \vdots \\
 & A_S x & & & & +B_S y_S & = b_S \\
 & x, y_0, y_1, y_2, \ldots, y_S \geq 0 & & & & &
\end{array}
$$

Challenge: - 1st Stage Variables (Here and Now) – Size of Deterministic Problem
- Scenarios Need to Capture Large Probability Spaces (e.g., Weather)
- Network Size, Time Horizon, Resolution

- **Existing Decomposition Approaches Converge Slowly** (Benders, Progressive Hedging)
  - Operations Need High Accuracy Solutions -Prices, Ensure Feasibility-

- **Alternative:** Exploit Linear Algebra Inside High-Efficiency Solvers (Scalable)

Parallel Stochastic Optimization

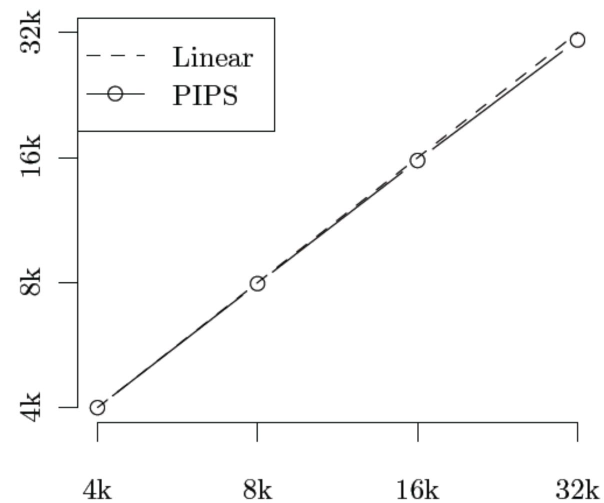
PIPS *Petra and Anitescu, 2010, Petra, Lubin, Anitescu and Z. 2011*

Interior-Point, Continuous, Coarse Decomposition

Based on OOQP *Gertz & Wright*, **Schur Complement-Based, Hybrid MPI/OpenMP**

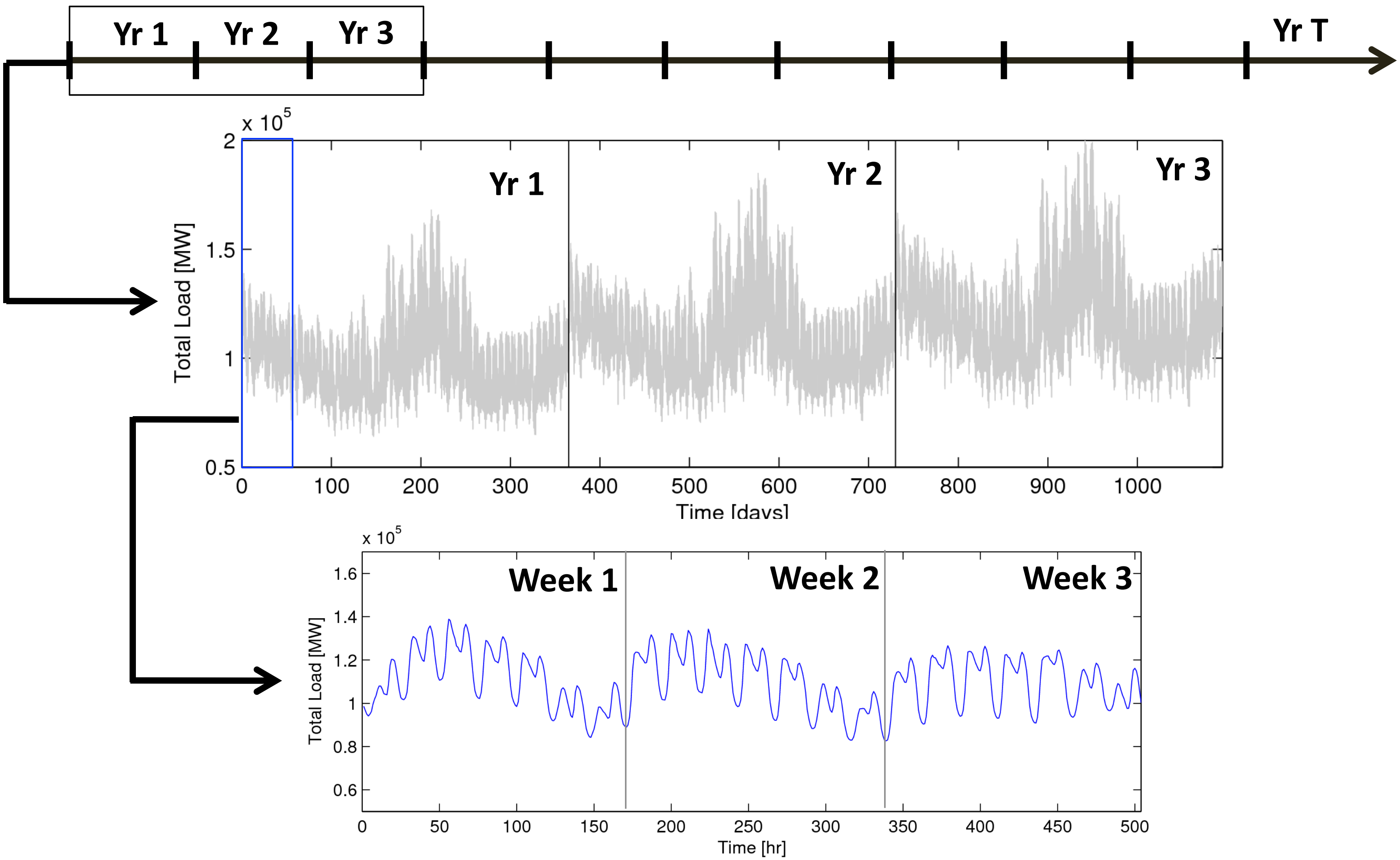
- **Test on Dispatch System on Illinois Grid with Rigorous Physical Model and Real Data**
- **O(10^4-10^5) Scenarios Needed to Cover High-Dimensional Spatio-Temporal Space over Wide Geographical Region**
- **6 Billion Variables Solved in Less than an Hour on BlueGene (128,000 Cores)**
- **O(10^5) First-Stage Variables – Parallel Dense Solver**
- **Finding: StochOpt Enables Integration of 20% Wind. Deterministic with Reserves Becomes Infeasible at 10%.**
- **Key Extensions:**
  - **Parallel Simplex Method**
  - **Couple with Parallel Branch & Bound for MILP**

Scaling on BlueGene/P



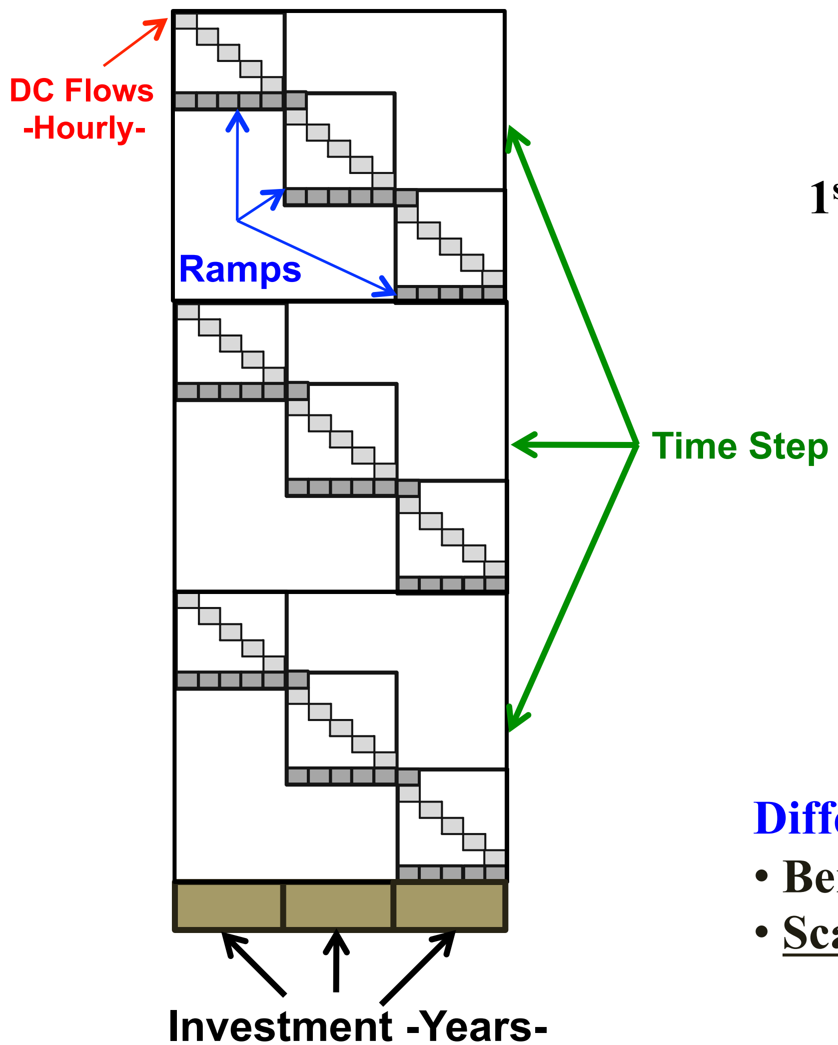
Stochastic Optimization for Expansion Planning

Capture Short Time-Scales in Multi-Year Planning



Market Volatility :: Constraints in Congestion and Ramping

Multi-Scale Structure



Two-Stage MILP

1st Stage: Investment **2nd Stage: Operations**

$$\min_{\mathbf{y}} \quad c^T\mathbf{y} + \frac{1}{S}\sum_{z\in\mathcal{Z}} Q_z(\mathbf{y})$$

$$\text{s.t.} \quad A\mathbf{y} \geq b, \ \boxed{\mathbf{y} \in \{0,1\}}$$

$$Q_z(\mathbf{y}) = \min_{\mathbf{x}_z} \ d_z^T\mathbf{x}_z$$

$$\text{s.t. } D_z\mathbf{x}_z \geq f_z - E_z\mathbf{y}, \ z \in \mathcal{Z}$$

Different Decomposition Alternatives

- **Benders, Linear Algebra, Progressive Hedging**
- **<u>Scalability</u> Analysis Needed**

Benders Decomposition

+ Decomposition at MILP Level
+ Exploits Existing Solvers: Branch & Bound and Linear Algebra (CPLEX, IPOPT)
- Slow Convergence
- Growing Size and Density in Master Problem

- At iteration $k = 0$, **start** with $LB^k = -\infty$, $UB^k = \infty$, gap $\epsilon > 0$.

- **Solve second-stage problem**:

$$\bar{Q}(\mathbf{y}^k) = \min_{\mathbf{x}} \ \bar{d}^T \bar{\mathbf{x}}, \text{ s.t. } \bar{D}\bar{\mathbf{x}} \geq \bar{f} - \bar{E}\mathbf{y}^k.$$

  – If solution $\mathbf{x}_*^k$ is **optimal**, define cut $L_\ell^*(\mathbf{y}) = (\bar{f} - \bar{E}\mathbf{y})^T \lambda_*^k$ and set $\ell \leftarrow \ell + 1$ and $UB_{k+1} = \min(UB_k, \bar{d}^T \bar{\mathbf{x}}_*^k + (\bar{f} - \bar{E}\mathbf{y}^k)^T \lambda_*^k)$.

  – If **infeasible**, define cut $L_\kappa^{inf}(\mathbf{y}) = (\bar{f} - \bar{E}\mathbf{y})^T \lambda^k$, and set $\kappa \leftarrow \kappa + 1$.

- **Solve the master problem:**

$$\begin{aligned} \min_{\mathbf{y},\theta} \quad & \theta \\ \text{s.t.} \quad & A\mathbf{y} \geq b \\ & \theta \geq c^T \mathbf{y} + L_j^*(\mathbf{y}), \ j = 0, ..., \ell \\ & L_i^{inf}(\mathbf{y}) \leq 0, \ i = 0, ..., \kappa, \end{aligned}$$

  to obtain $\mathbf{y}_*^k, \theta_*^k$ set $LB_{k+1} \leftarrow \theta_*^k$.

Termination Criterion

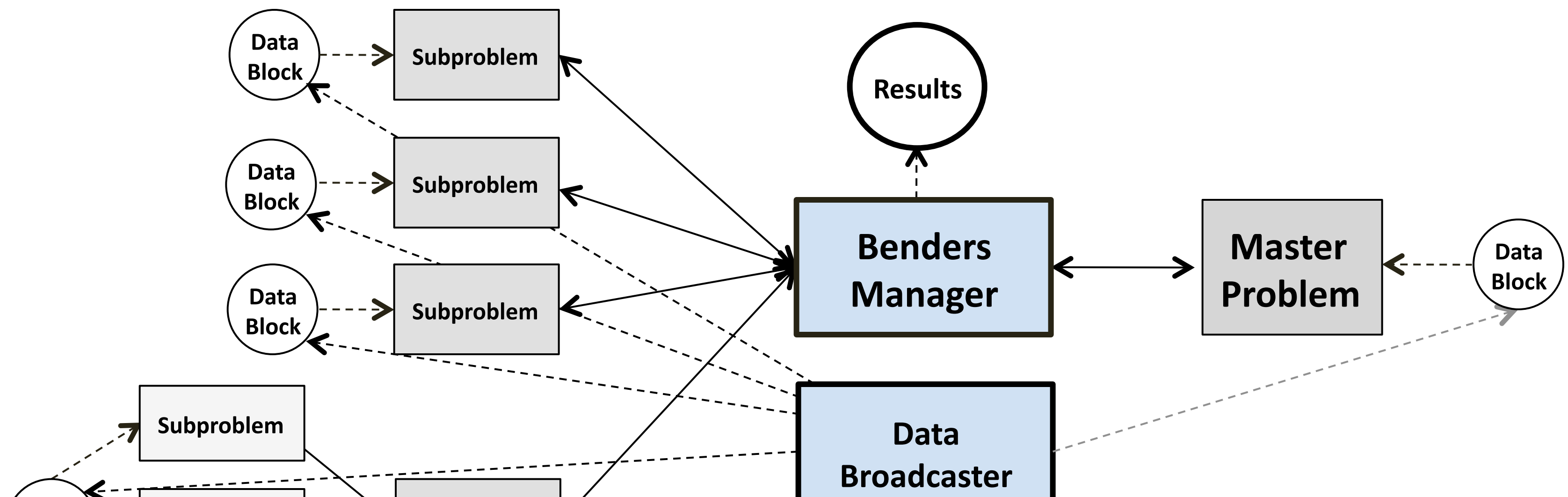
- If $(UB_{k+1} - LB_{k+1}) \leq \epsilon$, **stop**. Otherwise, set $\mathbf{y}_{k+1} \leftarrow \mathbf{y}_k, k \leftarrow k + 1$ and go back to Step 2.

Benders Framework

- **parBenders** : C++, MPI, OpenMP, GAMS *Xie, Leyffer & Z. 2010*

  Different Master and Subproblem Formulations

  Parallel Data & Model Management and <u>Reuse</u> – Minimize <u>Latency</u>



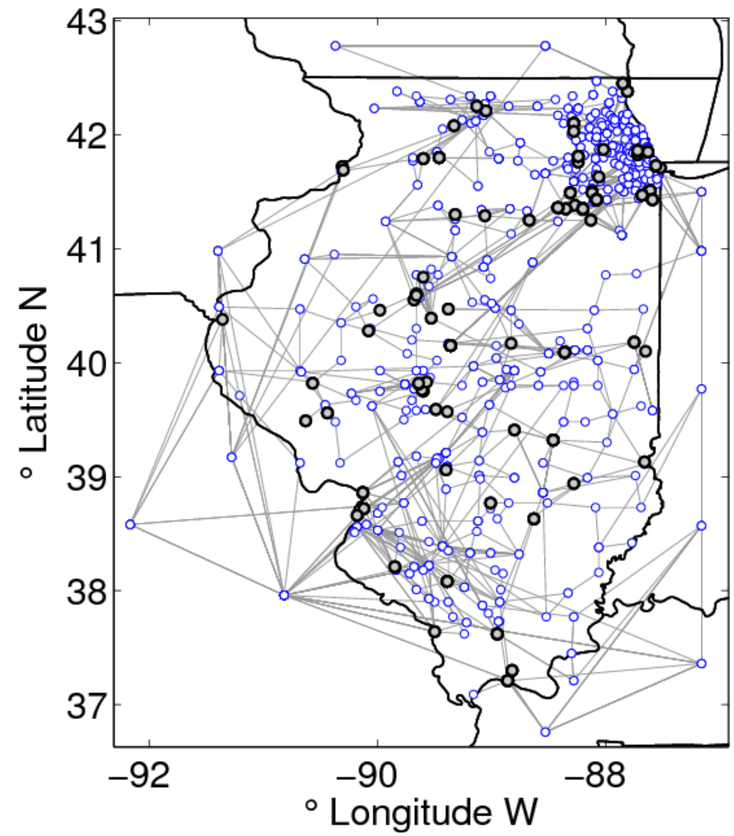
Case Study

Benchmark System –Illinois- Expansion Planning Under 30% Out of State Wind

Time Steps	Integers	Continuous	Constraints
1	100	4272	4009
5	100	21360	20045
10	100	42720	40090
50	100	213600	200450
100	100	427200	400900
200	100	854400	801800

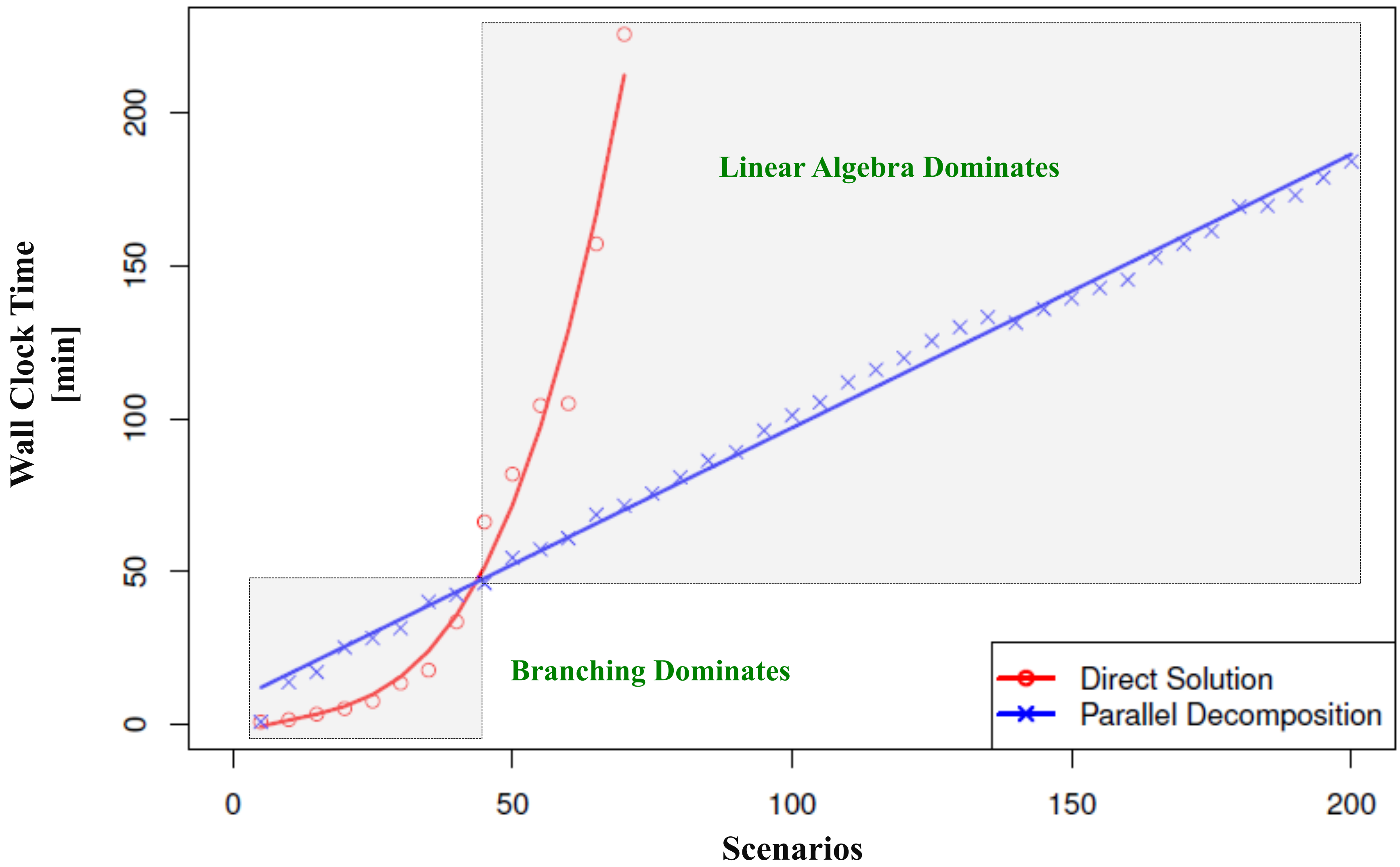
Shared-Memory Variant

16-Core Processor @2.27 GHz and with 24 Gb of RAM



Case Study

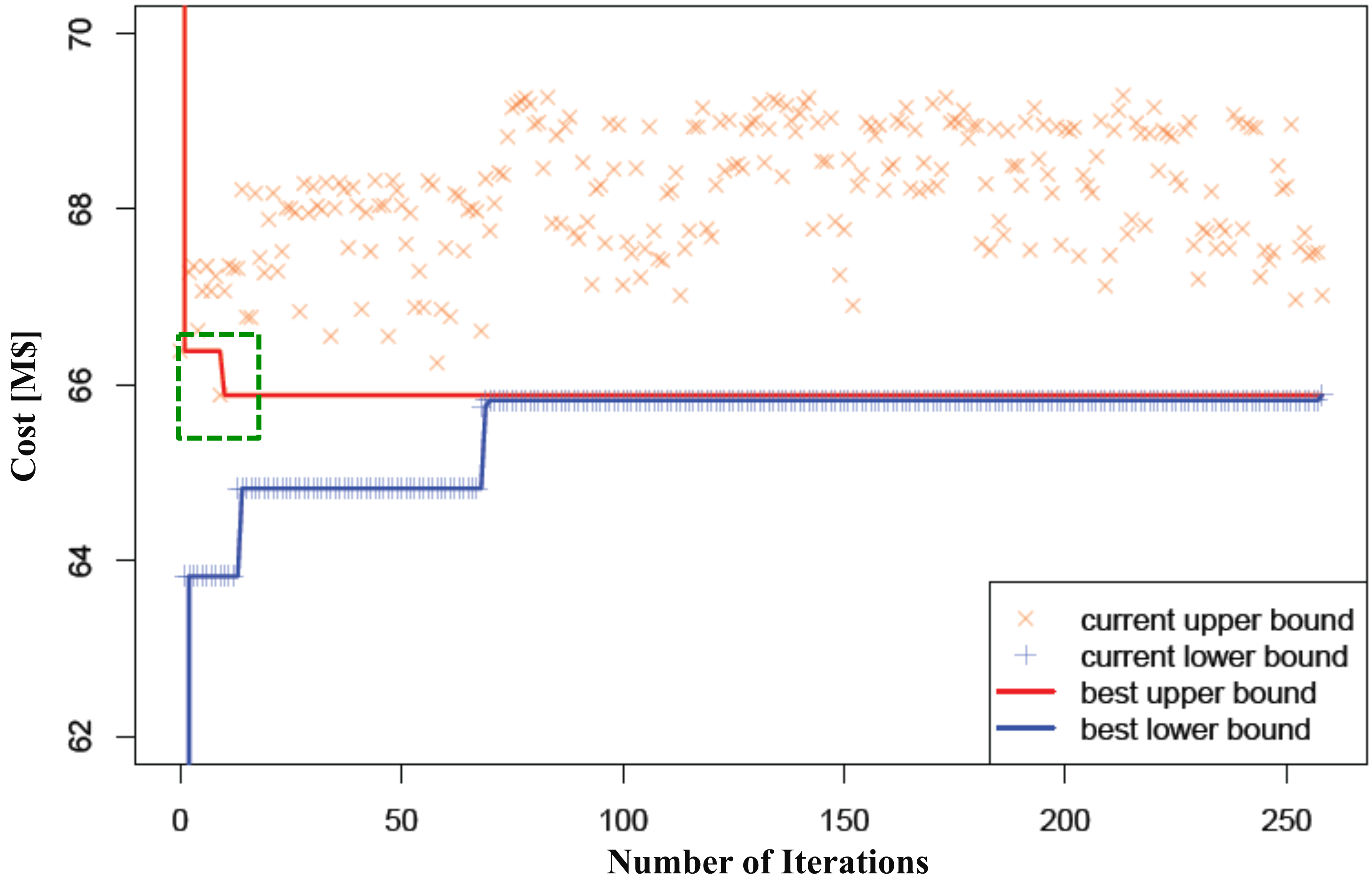
Scaling



Expansion Savings: <u>~1 Billion$/Yr</u> :: Enables Efficient Wind Adoption

Case Study

Convergence

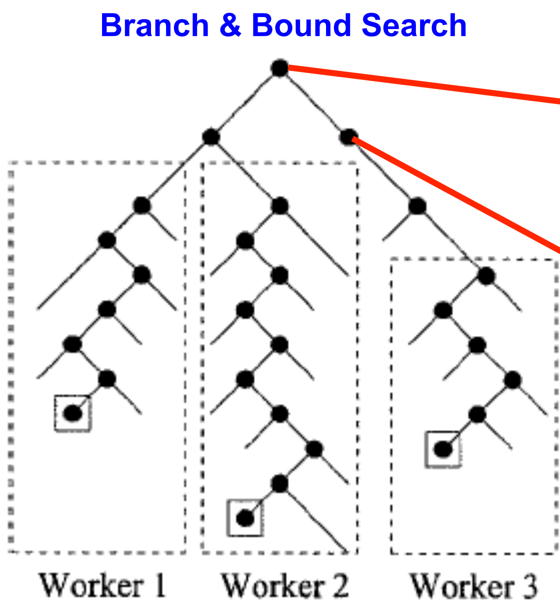


Solution Reached After 10 Iterations but Not Identified by Termination Criterion

Accuracy Less Critical in Planning :: Significant Savings in Few Iterations

Alternative Benders Strategy

- **Benders Decomposition at <u>LP Level</u>**



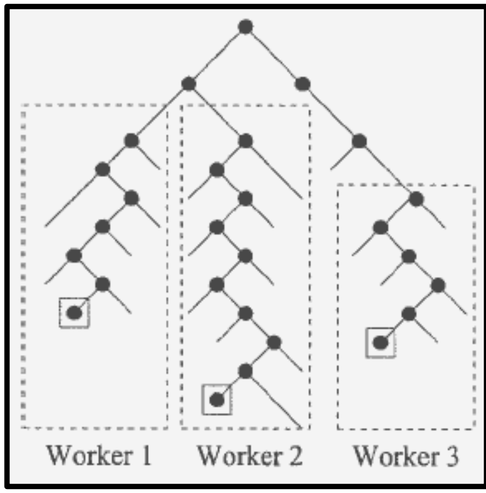
$$\begin{aligned} \min_{\mathbf{y}^0} \quad & c^T \mathbf{y}^0 + \frac{1}{S} \sum_{z \in \mathcal{Z}} Q_z(\mathbf{y}^0) \\ \text{s.t.} \quad & A\mathbf{y}^0 \geq b, \boxed{0 \leq \mathbf{y}^0 \leq 1} \\ & Q_z(\mathbf{y}^0) = \min_{\mathbf{x}_z} \; d_z^T \mathbf{x}_z \\ & \qquad \text{s.t. } D_z \mathbf{x}_z \geq f_z - E_z \mathbf{y}^0, \;\; z \in \mathcal{Z} \end{aligned}$$

$$\begin{aligned} \min_{\mathbf{y}^0} \quad & c^T \mathbf{y}^0 + \frac{1}{S} \sum_{z \in \mathcal{Z}} Q_z(\mathbf{y}^0) \\ \text{s.t.} \quad & A\mathbf{y}^0 \geq b, \boxed{0 \leq \mathbf{y}^0 \leq 1, \Pi \mathbf{y}^0 = 0, 1} \\ & Q_z(\mathbf{y}^0) = \min_{\mathbf{x}_z} \; d_z^T \mathbf{x}_z \\ & \qquad \text{s.t. } D_z \mathbf{x}_z \geq f_z - E_z \mathbf{y}^0, \;\; z \in \mathcal{Z} \end{aligned}$$

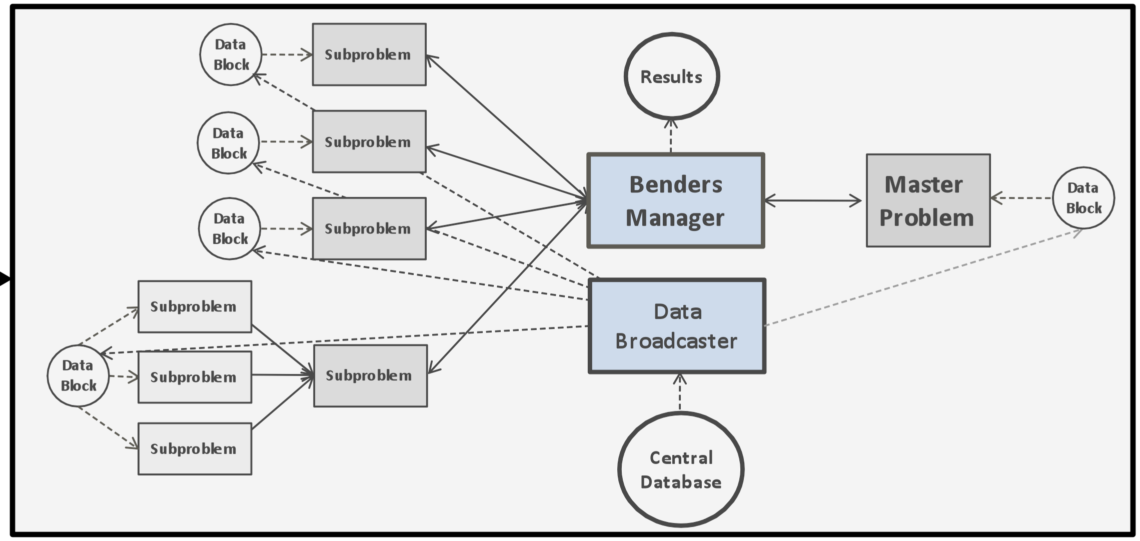
Advantages: **-KKT Error as Termination Criterion of Benders** **-Early Optimality Detection-**
-Warm-Start LPs Between Nodes
-Parallelize Branch & Bound Tree and Decomposition **-Minimize Latency-**
-Can use Other Parallel LP Strategies: **Bundle, Interior-Point**

New Benders Implementation

MINOTAUR Branch&Bound



parBenders



Largest LP Solved: - 2,000 Scenarios – 100 Integer, 8×10^6 Continuous
- Distributed Memory - MPI
- 74 Iterations, Solution Time 2 Min (Cold Start), 200 Cores

Pending:
- MILP Testing with Double Parallelization
- LP Warm-Starts
- BlueGene Testing –Less Memory/Node-

4. Dynamic Market Stability

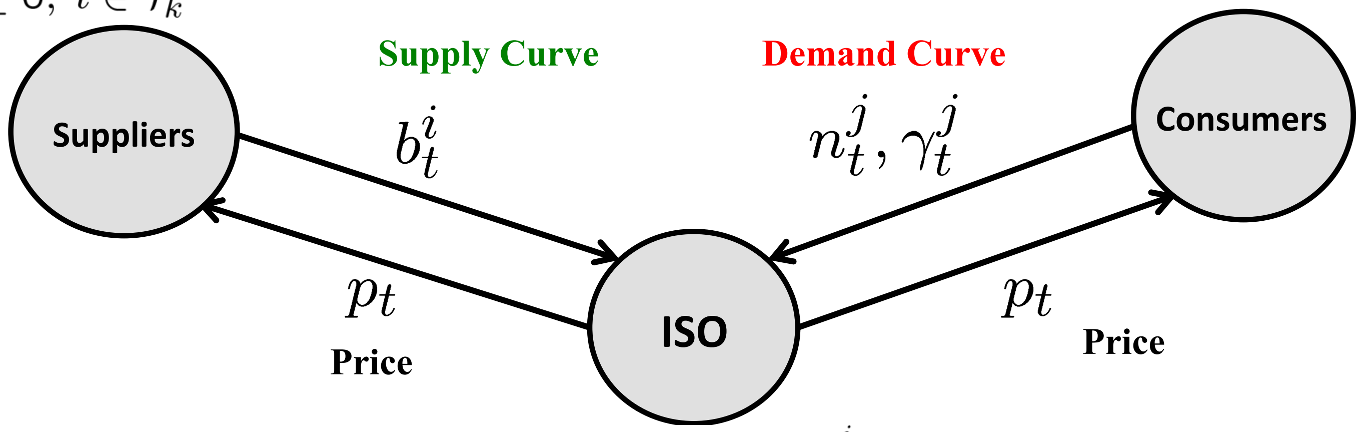
Market Game

$$\max_{b_t^i, \Delta b_t^i} \sum_{t \in \mathcal{T}_k} \left(p_t \cdot b_t^i \cdot p_t - c_t^i \left(b_t^i \cdot p_t \right) \right)$$

$$\text{s.t. } \underline{q}^i \le b_t^i \cdot p_t \le \overline{q}^i, \ t \in \mathcal{T}_k$$

$$b_t^i \ge 0, \ t \in \mathcal{T}_k$$

$$d_t^j = n_t^j - \gamma_t^j \cdot p_t$$



$$\min_{q_t^i, \Delta q_t^i} \sum_{t \in \mathcal{T}_k} \varphi_t := \sum_{t \in \mathcal{T}_k} \sum_{i \in \mathcal{S}} \int_0^{q_t^i} p_t(q, b_t^i) dq$$

$$\text{s.t. } q_{t+1}^i = q_t^i + \Delta q_t^i, \ i \in \mathcal{S}, t \in \mathcal{T}_k^-$$

$$\sum_{i \in \mathcal{S}} q_t^i \ge \sum_{j \in \mathcal{C}} d_t^j, \ t \in \mathcal{T}_k \qquad (p_t)$$

$$-\underline{r}^i \le \Delta q_t^i \le \overline{r}^i, \ i \in \mathcal{S}, t \in \mathcal{T}_k^-$$

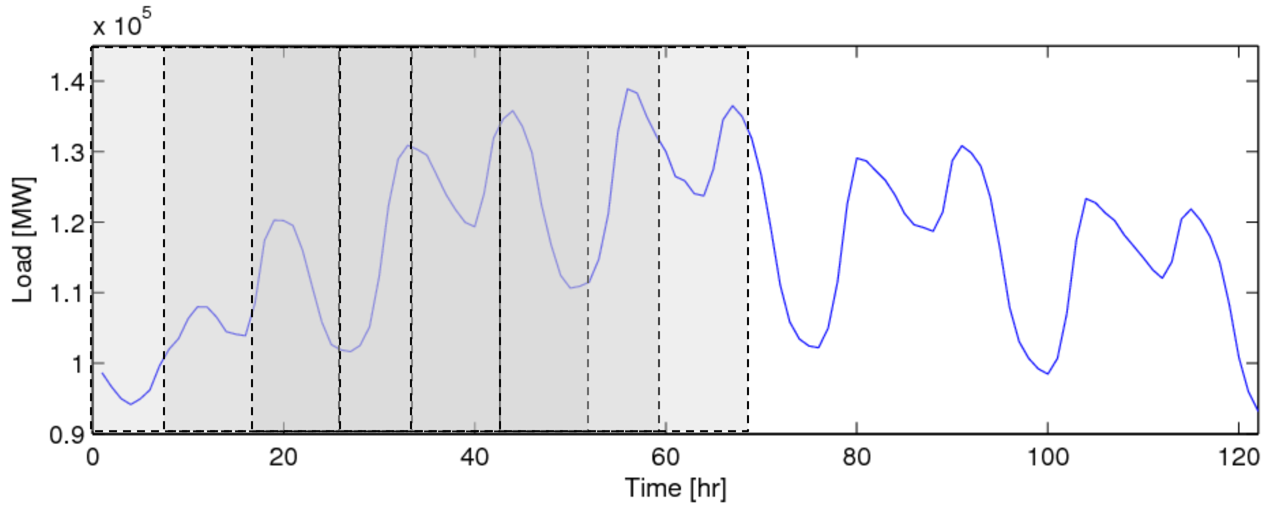
$$\underline{q}^i \le q_t^i \le \overline{q}^i, \ i \in \mathcal{S}, t \in \mathcal{T}_k$$

$$q_k^i = \text{given}, \ i \in \mathcal{S}.$$

Existing Design : **Game Runs Incompletely -Jacobi-Like Iteration-, No Notion of Stability**

Market Game

Current Markets: Game Implemented Over Receding Horizon



At k solve over $\mathcal{T}_k = \{k, ..., k+T\} \Rightarrow$ Implement Price p_k

At $k+1$ solve over $\mathcal{T}_{k+1} = \{k+1, ..., k+1+T\} \Rightarrow$ Implement Price p_{k+1}

Key Issues:

- How to Measure Dynamic Stability?

- Stability Under Finite Horizons

- Stability Under Incomplete Gaming

- Effect of Market Design: Frequency, Horizon, Stabilizing Constraints

Market Stability (A Proposal)

Constrained Market Clearing

$$\min_{q_t^i,\Delta q_t^i} \sum_{t\in\mathcal{T}_k} \varphi_t := \sum_{t\in\mathcal{T}_k}\sum_{i\in\mathcal{S}} \int_0^{q_t^i} p_t(q,b_t^i)dq$$

$$\text{s.t. } q_{t+1}^i = q_t^i + \Delta q_t^i,\ i\in\mathcal{S}, t\in\mathcal{T}_k^-$$

$$\sum_{i\in\mathcal{S}} q_t^i \geq \sum_{j\in\mathcal{C}} d_t^j,\ t\in\mathcal{T}_k \qquad (p_t)$$

$$-\underline{r}^i \leq \Delta q_t^i \leq \overline{r}^i,\ i\in\mathcal{S}, t\in\mathcal{T}_k^-$$

$$\underline{q}^i \leq q_t^i \leq \overline{q}^i,\ i\in\mathcal{S}, t\in\mathcal{T}_k$$

$$q_k^i = \text{given},\ i\in\mathcal{S}.$$

Unconstrained Market Clearing
(Utopia)

$$\min_{q_t^i} \sum_{t\in\mathcal{T}_k} \varphi_t = \sum_{t\in\mathcal{T}_k}\sum_{i\in\mathcal{S}} \int_0^{q_t^i} p_t(q,b_t^i)dq$$

$$\text{s.t. } \sum_{i\in\mathcal{S}} q_t^i \geq \sum_{j\in\mathcal{C}} d_t^j,\ t\in\mathcal{T}_k \qquad (\bar{p}_t)$$

$$\underline{q}^i \leq q_t^i \leq \overline{q}^i,\ i\in\mathcal{S}, t\in\mathcal{T}_k,$$

Property: **For Fixed** b_t^i **,** $\bar{\varphi}_t \leq \varphi_t, \forall t\in\mathcal{T}_k$

Definition: Market Efficiency. $\eta_t = \frac{\bar{\varphi}_t}{\varphi_t} \in [0,1]$

Definition: Market Stability. The market given by the ISO/Supplier/Consumer game is stable if, given $\eta_0 \in \{\eta \,|\, \eta \geq \epsilon\}$ **we have generation and demand sequences such that** $\eta_t \in \{\eta \,|\, \eta \geq \epsilon\},\ \forall t.$

Lyapunov Stability

Lyapunov Function = **<u>Indicator Function</u> (Sufficient Conditions, Compare Designs)**

Definition: Market Summarizing State.

$$\delta_{t+1} = \alpha(\eta_{t+1}, \epsilon) \cdot \delta_t \text{ with } \alpha(\eta, \epsilon) \leq 1 \text{ iff } \eta \leq \epsilon.$$

Observations: - Market Stability Implies Stability of Origin for Summarizing State

Abstract ISO Clearing Problem:

$$\begin{aligned} \min_{u_{\mathcal{T}_k^-}} \ & \sum_{t \in \mathcal{T}_k^-} (\delta_{t+1} - \delta_t) \\ \text{s.t. } & u_{\mathcal{T}_k} \in \Omega(\delta_k, d_{\mathcal{T}_k}) \\ & \delta_{t+1} = \alpha(\eta_{t+1}, \epsilon) \cdot \delta_t, \ t \in \mathcal{T}_k^- \\ & \eta_t \geq \epsilon, \ t \in \mathcal{T}_k \longleftarrow \textbf{ISO Stabilizing Constraint} \\ & \delta_k = \text{given.} \end{aligned}$$

Candidate Lyapunov Function

$$V_T(\delta_k, d_{\mathcal{T}_k}) := -\sum_{t \in \mathcal{T}_k^-} (\delta_{t+1} - \delta_t) = \delta_k - \delta_{k+T}.$$

Lyapunov Stability

Infinite Horizon: **If game with horizon $T = \infty$ is feasible then, the market is stable.**

Proof:

$$\begin{aligned}
\Delta V_T(\delta_k) &= V_\infty(\delta_{k+1}, m_{\mathcal{T}_{k+1}}) - V_\infty(\delta_k, m_{\mathcal{T}_k}) \\
&= \sum_{t=k+1}^{\infty} (\delta_t^{k+1} - \delta_{t+1}^{k+1}) - \sum_{t=k}^{\infty} (\delta_t^k - \delta_{t+1}^k) \\
&= \left(\delta_{k+1} - \delta_\infty^{k+1}\right) - \left(\delta_k - \delta_\infty^k\right) \\
&= -\left(\delta_k - \delta_{k+1}\right) \\
&= (\alpha(\eta_{k+1}, \epsilon) - 1) \cdot \delta_k \\
&\leq 0
\end{aligned}$$

Finite Horizon: **Define Terminal Cost,**

$$\Xi_k^1 := |V_T(\delta_{k+1}, m_{\mathcal{T}_{k+1}}) - V_{T-1}(\delta_{k+1}, m_{\mathcal{T}_k})|, \ \Xi_k^1 \to 0, \ T \to \infty$$

Finite Horizon: **If game with horizon $T < \infty$ is feasible and the terminal cost is bounded by accumulation term, then the market is stable.**

Proof:

$$\begin{aligned}
\Delta V_T(\delta_k) &= V_T(\delta_{k+1}, m_{\mathcal{T}_{k+1}}) - V_T(\delta_k, m_{\mathcal{T}_k}) \\
&= (\alpha(\eta_{k+1}, \epsilon) - 1) \cdot \delta_k + \Xi_k^1 \\
&\leq 0
\end{aligned}$$

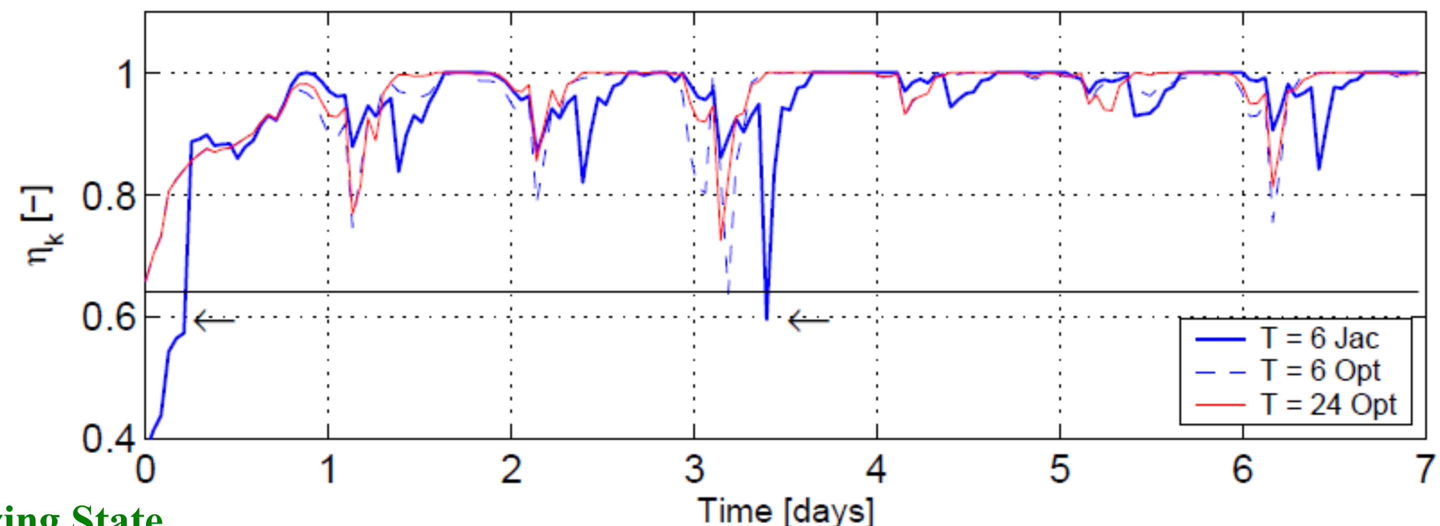
Key Insights:
- **Incomplete Game Cannot be Guaranteed to be Stable**
- **Stabilizing ISO Constraint “Filters Out” Suboptimal Bids :: Manipulation**
- **Stability Strongly Affected by Forecast Horizon**

Stability

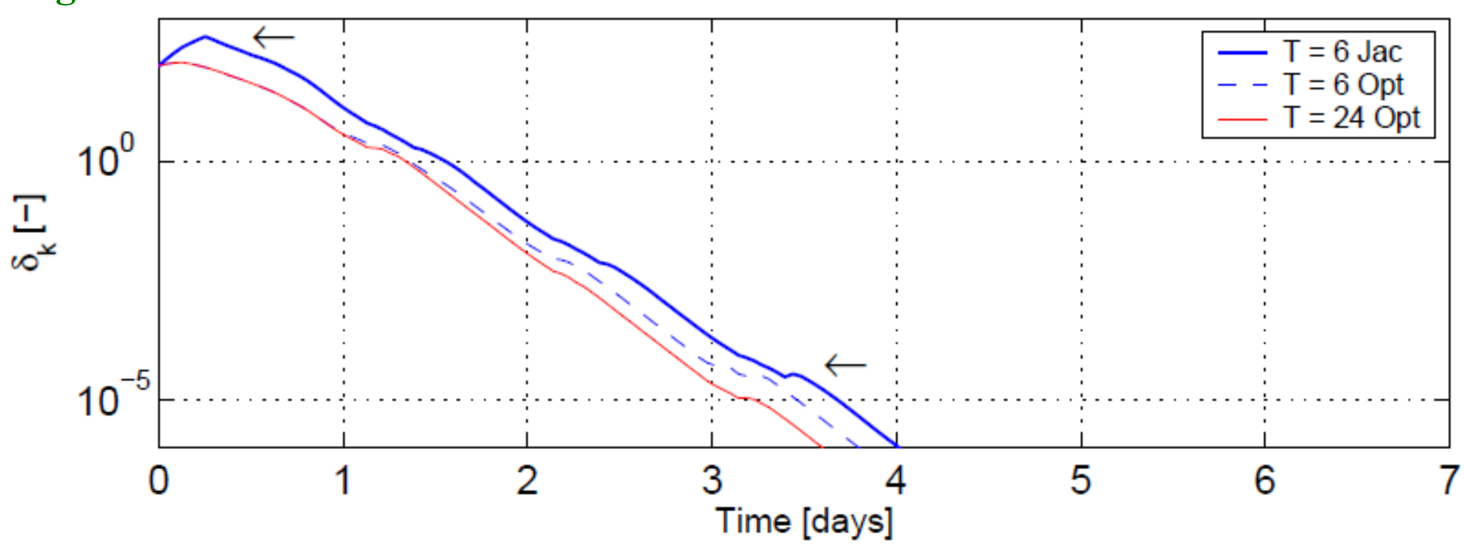
Consider 3 Market Designs

- 6 Hours Horizon, Incomplete Gaming (Jac)
- 6 Hours Horizon, Complete Gaming (Opt)
- 24 Hours Horizon, Complete Gaming (Opt)

Efficiency



Summarizing State

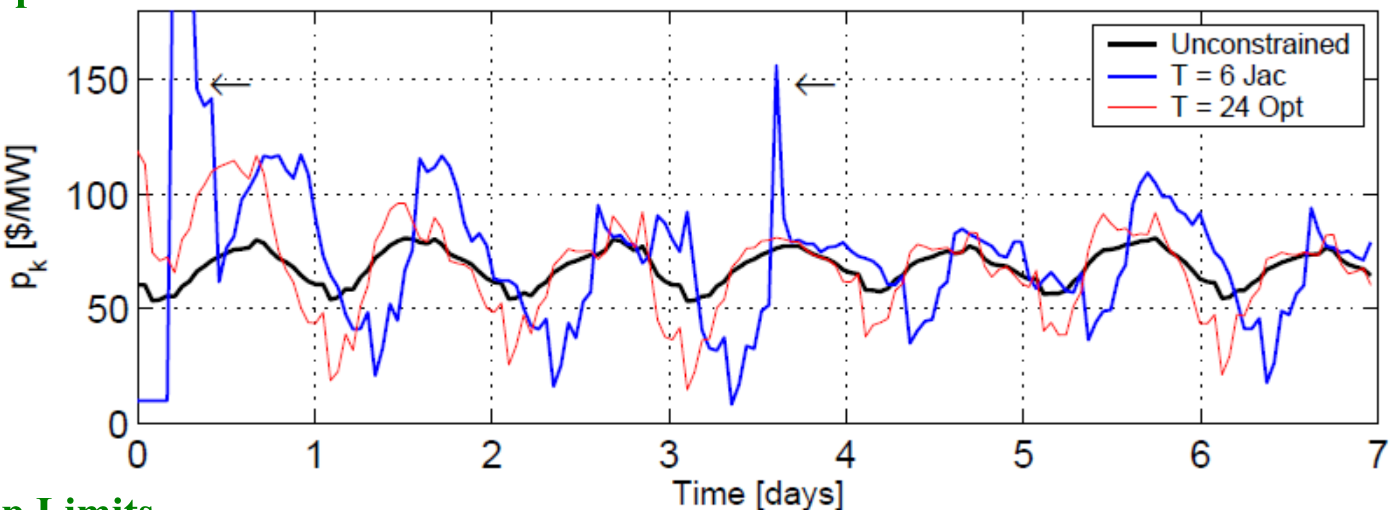


Stability

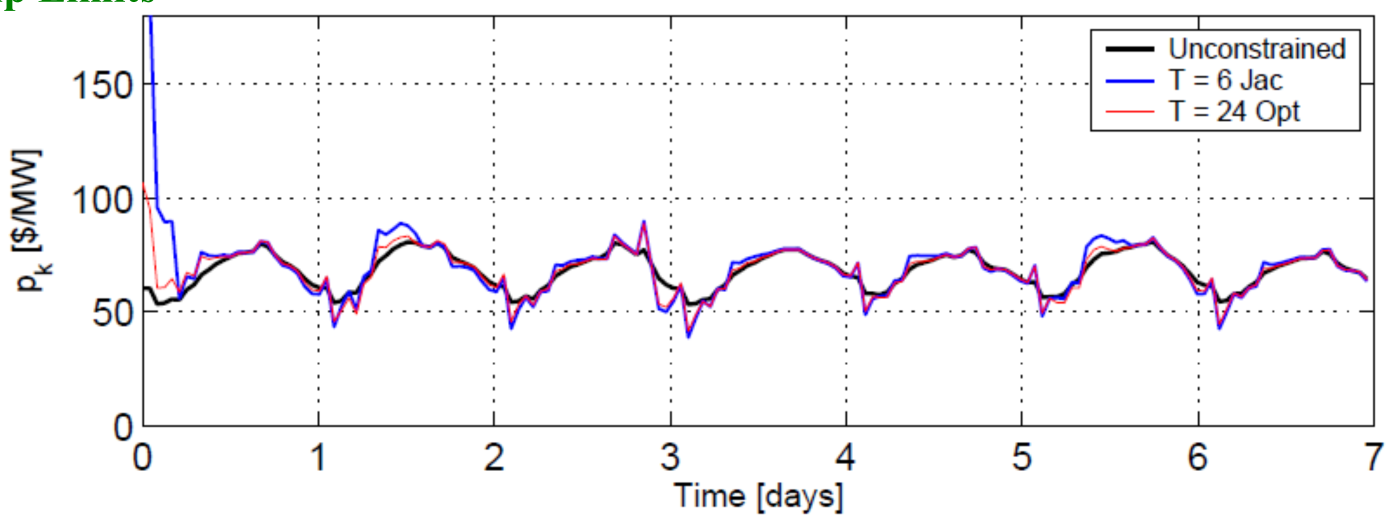
Consider 3 Market Designs

- 6 Hours Horizon, Incomplete Gaming (Jac)
- 6 Hours Horizon, Complete Gaming (Opt)
- 24 Hours Horizon, Complete Gaming (Opt)

Tight Ramp Limits



Lose Ramp Limits



5. Conclusions and Open Questions

Conclusions and Open Questions

Market Volatility Induced by Computational Limitations and Market Design

- Anticipation :: Forecast Horizon, Stochastic Optimization
- Lack of Stabilizing Mechanism in ISO Clearing
- Limited Ramping and Transmission Capacity

Argonne's Vision: Fully-Integrated Expansion Planning with Detailed Market Behavior

- Incorporate Detailed Physical Models
- Capture Multiple Scales
- Incorporate Uncertainty and Risk
- Leverage Available High-Performance Computing Capabilities

Research Needed:

- Scalable Methods for MILP and LP/QP (Decomposition, Linear Algebra)
- Capture Dynamic Effects (Market and Cascading Failures)
- Dynamic Market Models and Monitoring

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Further Reading

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On the Convergence of Day-Ahead and Real-Time Electricity Markets

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FERC
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Expansion Planning Formulation

Investment -First Stage- **Economic Surplus -Second Stage-**

$$\min \sum_{t\in\mathcal{T}} \sum_{(i,j)\in\mathcal{L}^C} c^L_{t,i,j}(\mathbf{y}^L_{t+1,i,j} - \mathbf{y}^L_{t,i,j}) + \sum_{t\in\mathcal{T}} \sum_{k\in\mathcal{K}} \sum_{j\in\mathcal{G}} c^G_{t,j} \cdot G_{t,k,j}$$

s.t.

Planning Constraints

$$\mathbf{y}^L_{t+1,i,j} \geq \mathbf{y}^L_{t,i,j},\ t \in \mathcal{T}, (i,j) \in \mathcal{L}_C$$

$$\mathbf{y}^L_{t,i,j} \in [0,1],\ t \in \mathcal{T}, (i,j) \in \mathcal{L}_C$$

Operational Constraints

$$|P_{t,k,i,j}| \leq P^{max}_{i,j} \cdot \mathbf{y}^L_{t,i,j},\ t \in \mathcal{T}, k \in \mathcal{K}, (i,j) \in \mathcal{L}_C$$

$$|P_{t,k,i,j} - b_{i,j}(\theta_{t,k,i} - \theta_{t,k,j})| \leq M_{i,j} \cdot (1 - \mathbf{y}^L_{t,i,j}), t \in \mathcal{T}, k \in \mathcal{K}, (i,j) \in \mathcal{L}_C$$

$$|P_{t,k,i,j}| \leq P^{max}_{i,j},\ t \in \mathcal{T}, k \in \mathcal{K}, (i,j) \in \mathcal{L}_I$$

$$P_{t,k,i,j} = b_{i,j}(\theta_{t,k,i} - \theta_{t,k,j}), t \in \mathcal{T}, k \in \mathcal{K}, (i,j) \in \mathcal{L}_I$$

$$\sum_{(i,j)\in\mathcal{L}_j} P_{t,k,i,j} + \sum_{i\in\mathcal{W}_j} L^W_{t,k,i} + \sum_{i\in\mathcal{G}_j} G_{t,k,i} = \sum_{i\in\mathcal{D}_j} L^D_{t,k,i},\ t, \in \mathcal{T}, k \in \mathcal{K}, j \in \mathcal{B}$$

$$0 \leq G_{t,k,j} \leq G^{max}_j,\ t \in \mathcal{T}, k \in \mathcal{K}, j \in \mathcal{G}$$

$$\underline{r}_j \leq G_{t,k+1,j} - G_{t,k,j} \leq \overline{r}_j,\ t \in \mathcal{T}, k \in \mathcal{K}, j \in \mathcal{G}$$ **Dynamic Ramps**

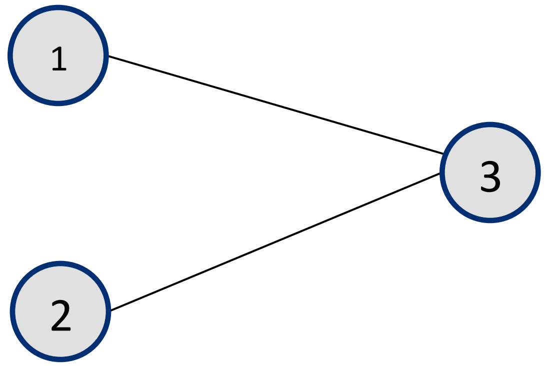
$$|\theta_{t,k,j}| \leq \theta^{max}_j,\ t \in \mathcal{T}, k \in \mathcal{K}, j \in \mathcal{B}.$$

MILP Size: **O(10^3-10^4) Integers, O(10^6-10^8) Continuous**

Avoid Simulation-Based Optimization – Not Scalable

Horizon and Ramp Constraints

$c_1 = 50\$/MW$
$r_1 = 1MW/hr$



$L_3^t = 28MW \rightarrow L_3^{t+1} = 26MW$

$c_2 = 200\$/MW$
$r_2 = 15MW/hr$

No Ramp Constraints

$$\lambda^t = 50\$/MW(28,0) \rightarrow \lambda^{t+1} = 50\$/MW(26,0)$$

Ramp Constraints (No Foresight)

$G_{t-1}^1 = 27MW$
$G_{t-1}^2 = 1MW$

$$\lambda^t = 50\$/MW(28,0) \rightarrow \lambda^{t+1} = 0\$/MW(27,0)$$

Ramp Constraints (No Foresight)

$G_{t-1}^1 = 26MW$
$G_{t-1}^2 = 2MW$

$$\lambda^t = 50\$/MW(27,1) \rightarrow \lambda^{t+1} = 50\$/MW(26,0)$$

Ramp Constraints (with Foresight)

$G_{t-1}^1 = 27MW$
$G_{t-1}^2 = 1MW$

$$\lambda^t = 55.35\$/MW(27,1) \rightarrow \lambda^{t+1} = 50\$/MW(26,0)$$

Ramps and Short Horizons Induce Volatility – Propagation In Time